

# Learning without search

Geoff Webb, Zijian Zheng, Kai Ming Ting, Zihai Wang, Fei Zheng, Janice Boughton,  
Houssam Salem

Monash University,  
Melbourne, Australia

<http://www.csse.monash.edu.au/~webb>

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- Averaged  $n$ -Dependence Estimators ( $A_nDE$ ) is a family of classification learning algorithms that exemplifies an alternative paradigm
  - learner uses a fixed model to extrapolate from observed low-order probabilities to the required high-order probability

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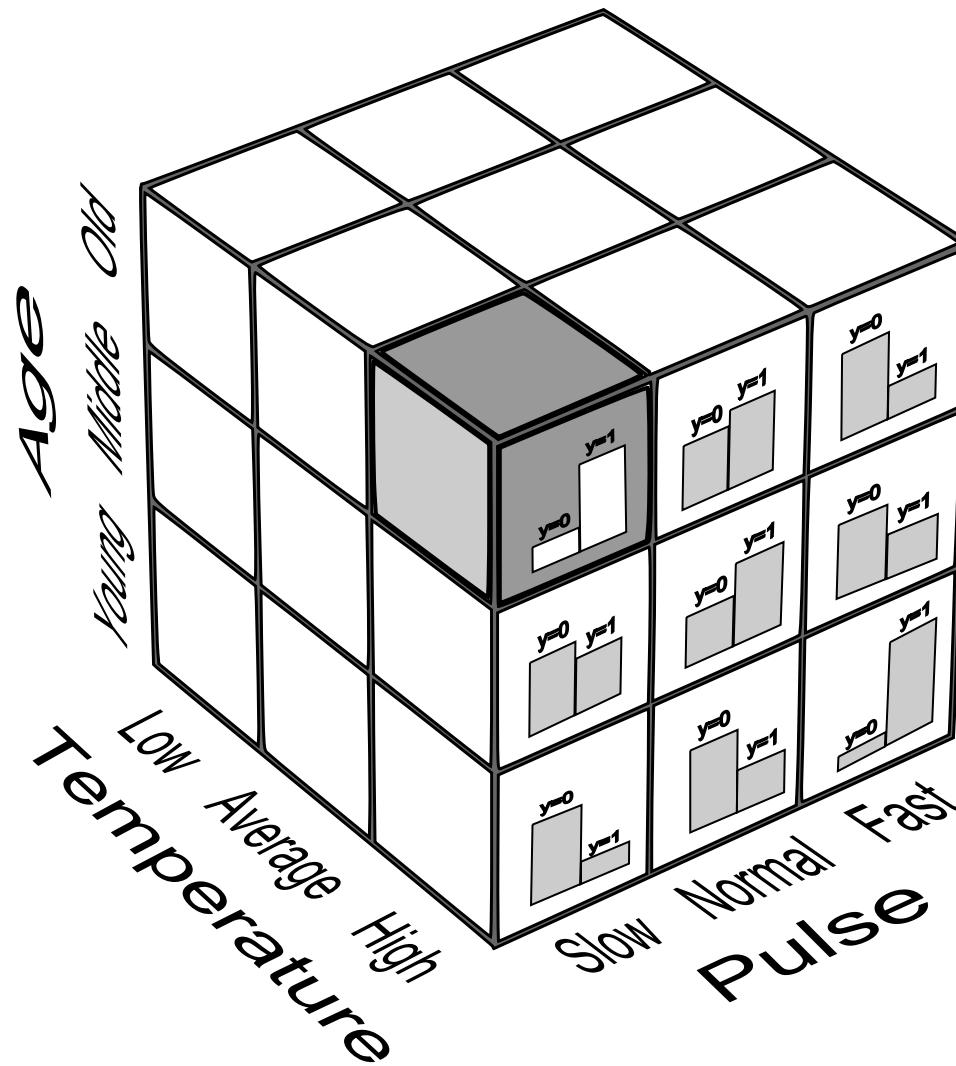


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  - alternative paradigms exist;  
*if things aren't going right ...*
- generative learning can achieve the same low bias profile as discriminative.
- Unique and valuable combination of practical features

*go left!*

# Classification: A geometric view



# Learning by extrapolation

- In contrast to search paradigm, naive Bayes extrapolates to high-order conditional probabilities from lower-order probabilities.

# Naive Bayes

- $$\begin{aligned} P(y | \mathbf{x}) &\propto P(y, \mathbf{x}) \\ &= P(y)P(\mathbf{x} | y) \end{aligned}$$

# Naive Bayes

- $P(y | \mathbf{x}) \propto P(y, \mathbf{x})$   
 $= P(y)P(\mathbf{x} | y)$
- Attribute independence assumption

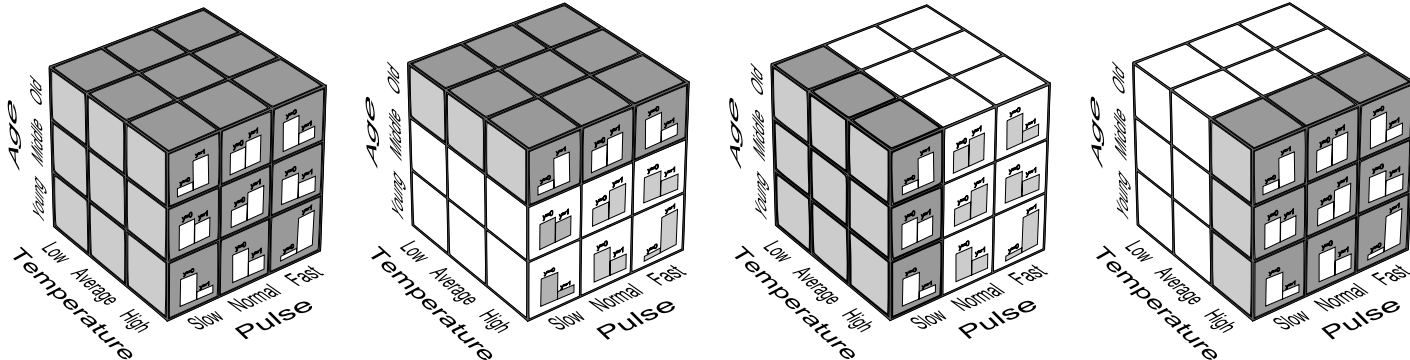
- $P(\mathbf{x} | y) = \prod_{i=1}^n P(x_i | y)$

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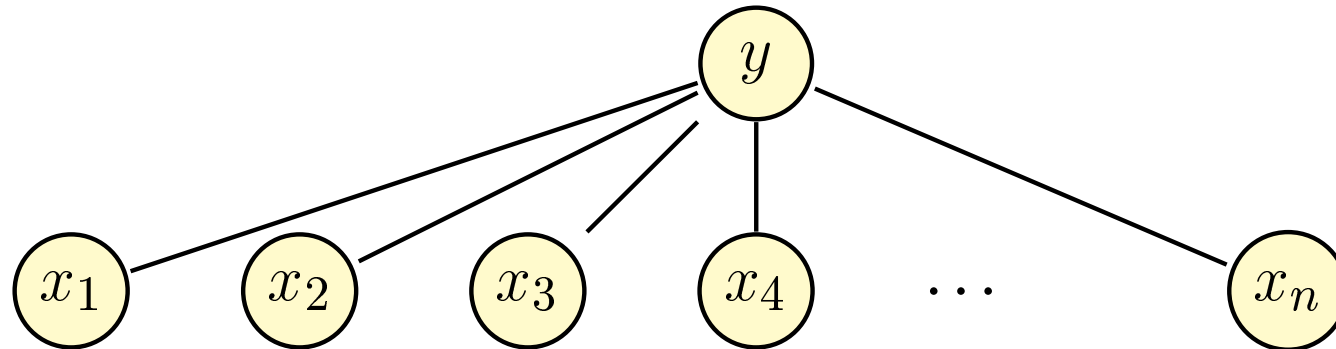
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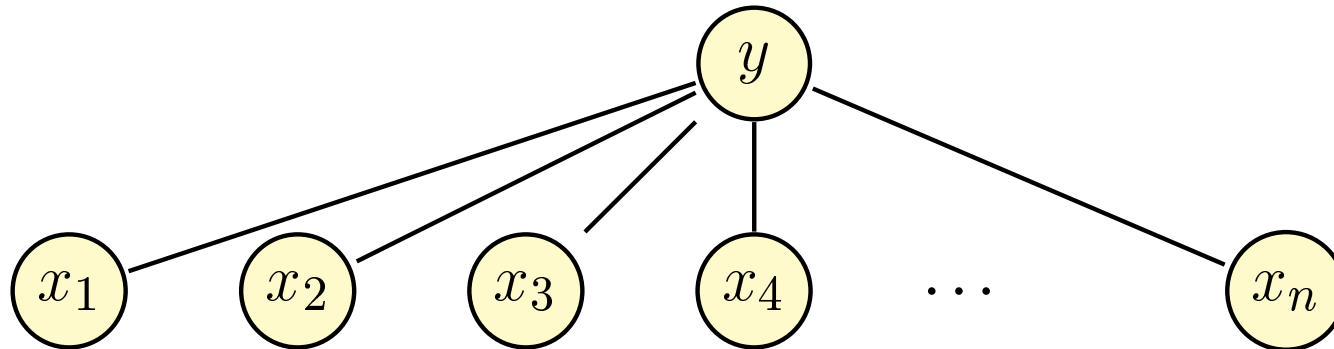
- No search
  - extrapolate high-order probabilities from low order probabilities  $P(y)$  and  $P(x_i | y)$



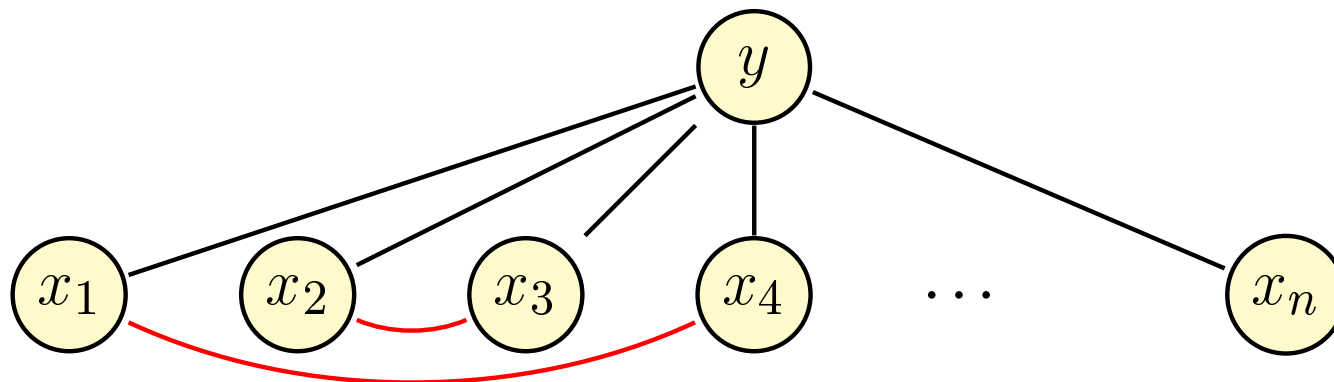
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- Adding arbitrary links will decrease bias but increase variance





# But how to decide which links?

- Could use search
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- Alternative: use all of a class of models and combine predictions

# ANDE

- Averaged  $n$  Dependence Estimators

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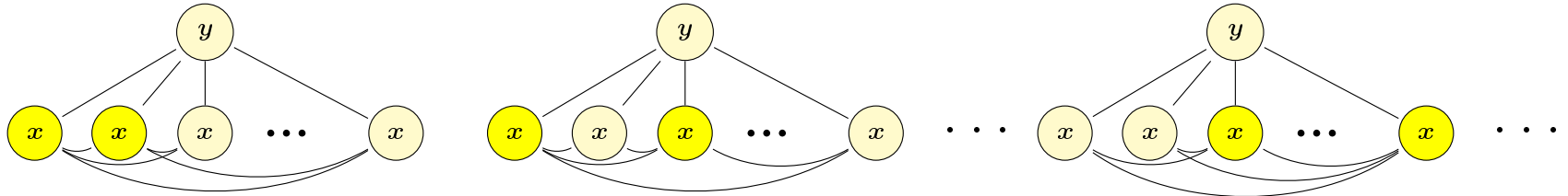
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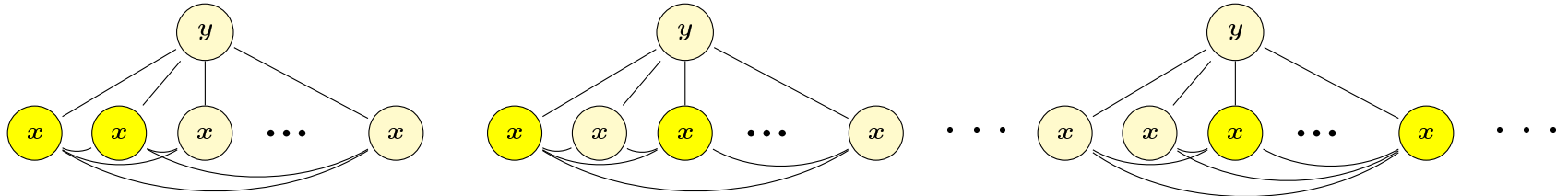
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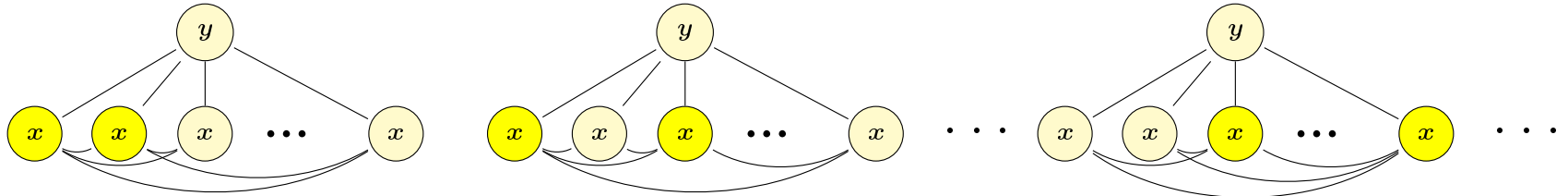
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- Each model has lower bias but higher variance than NB
- Ensembling reduces the variance



# ANDE (derivation)

- ANDE aims to use

$$AnDE(y, \mathbf{x}) = \sum_{s \in S^n} P(y, x_s) P(\mathbf{x} | y, x_s) / \binom{a}{n}.$$

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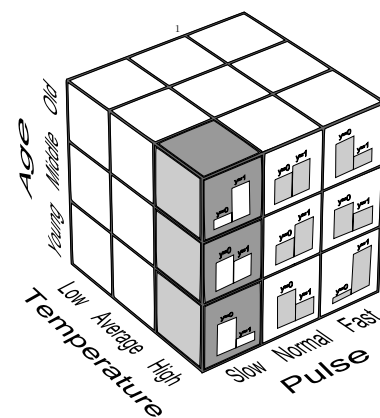
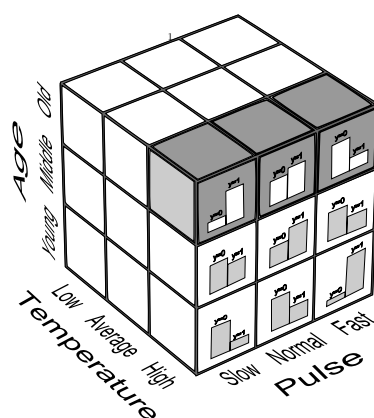
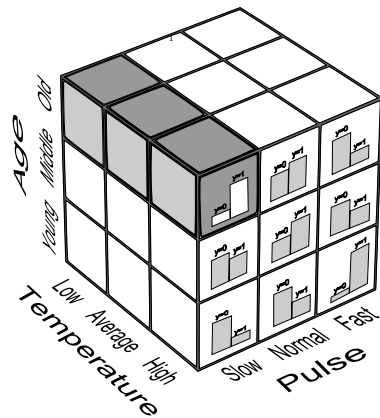
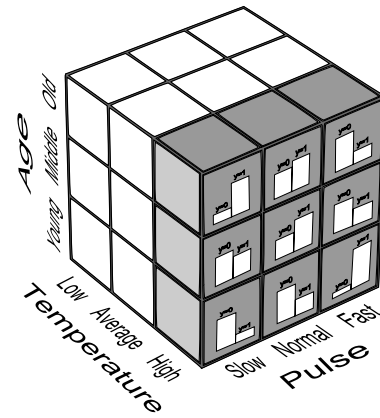
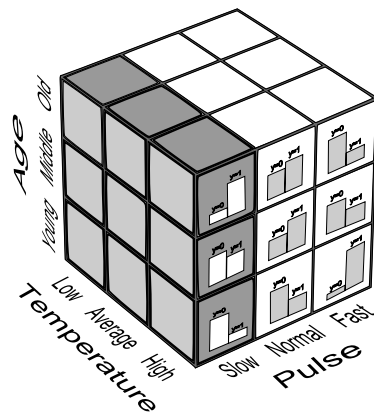
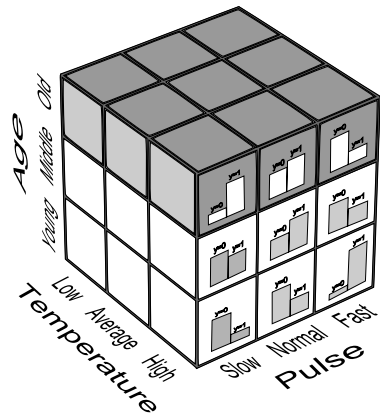
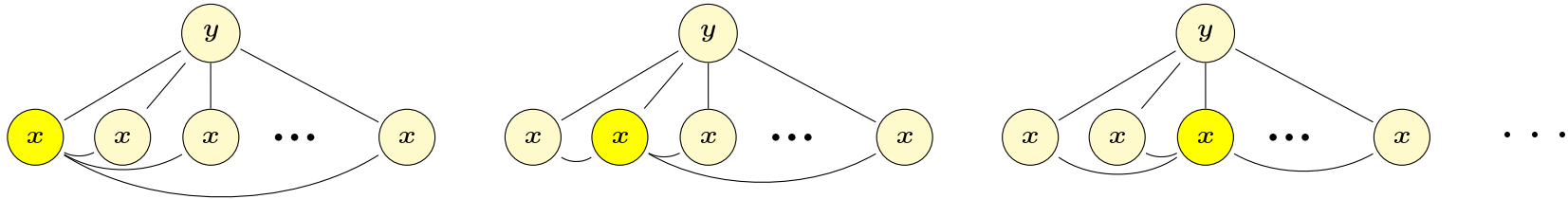
- In practice we use

$$AnDE(y, \mathbf{x}) = \begin{cases} \frac{\sum_{s \in S^n} \delta(s) P(y, x_s) P(\mathbf{x} | y, x_s)}{\sum_{s \in S^n} \delta(s)} & : \sum_{s \in S^n} \delta(s) > 0 \\ A(n-1)DE(y, \mathbf{x}) & : \text{otherwise} \end{cases}$$

# ANDE Equivalences

- $A0DE = NB$
- $A1DE = AODE$

# AODE



# Popular

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- has asymptotic error of the Bayes optimal classifier!
- has computational complexity of at least  $O(k \prod_{i=1}^a v_i)$

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- In practice our Weka implementation of A3DE is defeated by high-dimensional data



# Evaluation

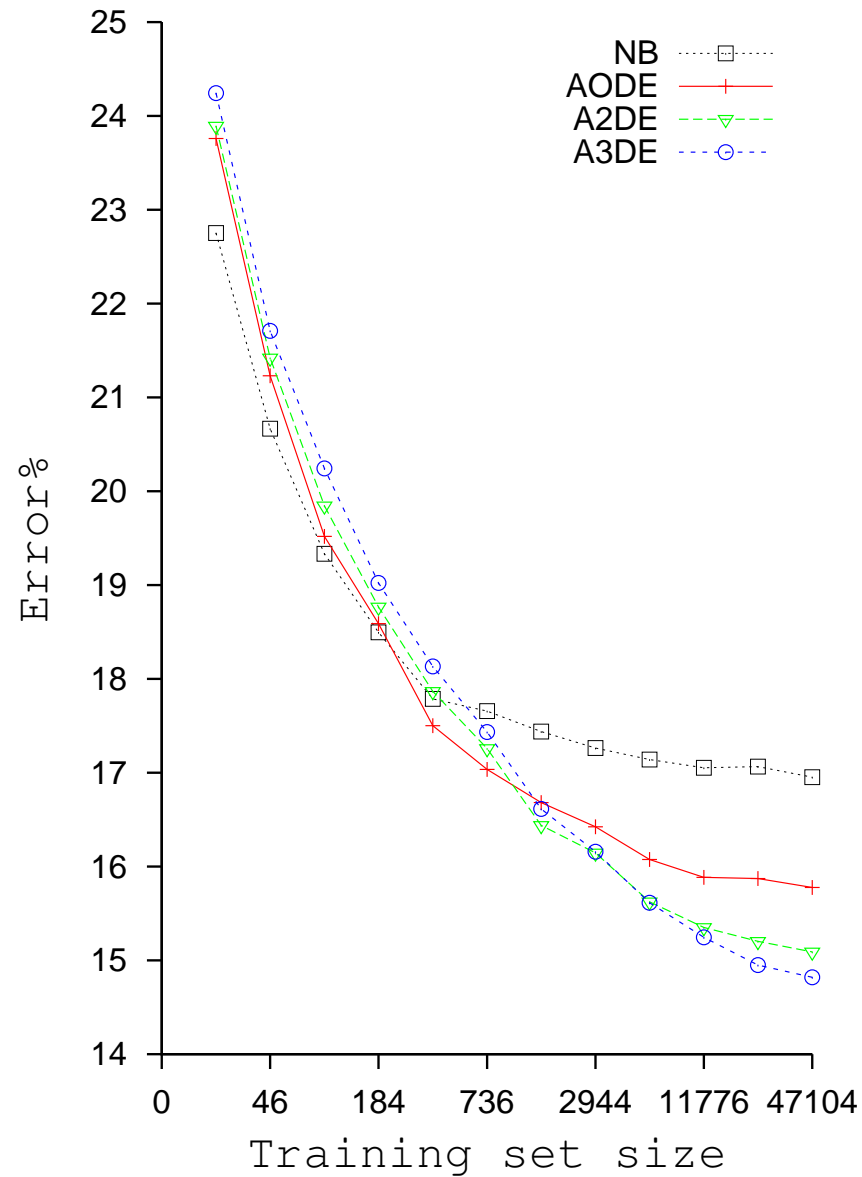
- 62 UCI data sets used previously in related research
- Use fifty runs of two-fold cross validation to estimate bias, variance, 0-1 loss and RMSE.

# ANDE, $n = 0, 1$ and $2$

Win/Draw/Loss

	A2DE vs AODE		A2DE vs NB		AODE vs NB	
	W/D/L	$p$	W/D/L	$p$	W/D/L	$p$
Bias	47/0/15	<0.001	49/2/11	<0.001	48/0/14	<0.001
Variance	19/1/42	<0.001	15/0/47	<0.001	20/1/41	0.005
0-1 loss	33/2/27	0.259	42/1/19	0.002	44/1/17	<0.001
RMSE	35/1/26	0.153	45/0/17	<0.001	49/1/12	<0.001

# Error as function of training set size



# Higher-order probabilities vs search

- AODE & A2DE vs TAN
- Win/Draw/Loss

	A2DE vs TAN		AODE vs TAN	
	W/D/L	$p$	W/D/L	$p$
Bias	34/0/28	0.263	20/1/41	<b>0.005</b>
Variance	48/0/14	<b>&lt;0.001</b>	52/1/9	<b>&lt;0.001</b>
Zero-one loss	48/0/14	<b>&lt;0.001</b>	43/1/18	<b>0.001</b>
RMSE	43/1/18	<b>0.001</b>	40/1/21	<b>0.010</b>

# Higher-order probabilities vs search

- A2DE & AODE vs MAPLMG
- Win/Draw/Loss

	A2DE vs MAPLMG		AODE vs MAPLMG	
	W/D/L	$p$	W/D/L	$p$
Bias	40/0/22	0.015	17/4/41	0.001
Variance	19/1/42	0.002	36/5/21	0.031
Zero-one loss	30/1/31	0.500	22/4/36	0.043
RMSE	34/1/28	0.263	19/0/39	0.006

- Win/Draw/Loss, A2DE vs MAPLMG on 10 largest data sets
  - 10/0/0,  $p = 0.001$

# No search vs state-of-the-art

- A2DE vs RF10, RF100
- Win/Draw/Loss

	A2DE vs RF10		A2DE vs RF100	
	W/D/L	$p$	W/D/L	$p$
Bias	14/1/47	<0.001	20/2/40	0.007
Variance	56/1/5	<0.001	46/1/15	<0.001
0-1 loss	40/1/21	0.010	34/2/26	0.399
RMSE	39/0/23	0.028	34/0/28	0.263

# Times

- Average training/test time per instance, excluding Census Income

	<b>NB</b>	<b>AODE</b>	<b>A2DE</b>	<b>TAN</b>	<b>MAPLMG</b>	<b>RF10</b>	<b>RF100</b>
Train	0.0005	0.0007	0.0413	0.0022	0.1290	0.0177	0.1645
Test	0.0001	0.0022	0.0552	0.0002	0.0025	0.0001	0.0017

# Scalability: training quantity

- On 10 smallest data sets

---

	<b>NB</b>	<b>AODE</b>	<b>A2DE</b>	<b>TAN</b>	<b>MAPLMG</b>	<b>RF10</b>	<b>RF100</b>
Train	0.0020	0.0020	0.0920	0.0064	0.1339	0.0114	0.0844

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- On 10 largest data sets, excluding Census Income

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	<b>NB</b>	<b>AODE</b>	<b>A2DE</b>	<b>TAN</b>	<b>MAPLMG</b>	<b>RF10</b>	<b>RF100</b>
Train	0.0001	0.0002	0.0077	0.0004	0.1360	0.0229	0.2016

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# Scalability: dimensionality

- On 10 lowest dimensional data sets (4-8 atts)

	<b>NB</b>	<b>AODE</b>	<b>A2DE</b>	<b>TAN</b>	<b>MAPLMG</b>	<b>RF10</b>	<b>RF100</b>
Train	0.0010	0.0009	0.0011	0.0018	0.0448	0.0046	0.0311
Test	0.0001	0.0002	0.0002	0.0001	0.0003	0.0001	0.0004

- On 10 highest dimensional data sets (43-70 atts)

	<b>NB</b>	<b>AODE</b>	<b>A2DE</b>	<b>TAN</b>	<b>MAPLMG</b>	<b>RF10</b>	<b>RF100</b>
Train	0.0008	0.0017	0.1870	0.0067	0.5025	0.0435	0.4125
Test	0.0002	0.0097	0.2976	0.0005	0.0097	0.0002	0.0033

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- Feating uses the mode of the posterior class predictions while ANDE uses the mean of the joint probability estimates
- ANDE has lower bias but higher variance than Feating NB

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- Understand why ensembling joint probabilities results in lower bias than ensembling posteriors

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  - direct theoretical basis (Bayes optimal prediction except insofar as clearly specified assumptions are violated).